Rocket City Math League 2004-2005

Apollo Solutions Round 2

1. Answer: **0**

$2^{x^2+2x-4} = (2^2)^x = 2^{2x}$

$x^2 + 2x - 4 = 2x$

$x^2 - 4 = 0$

Sum of solutions = $\frac{-b}{a} = \frac{-0}{1} = 0$

2. Answer: **[–7, –5) U (–5, ∞)**

As a fraction, the denominator (x + 5) cannot by 0 $\Rightarrow x \neq -5$

The sum under the radical ($\sqrt{x+7}$) cannot be < 0 $\Rightarrow x \geq -7$

Domain: [–7, –5) U (–5, ∞)

3. Answer: **3**

Albert: $\sin^2 A + \cos^2 B = 1$

$\sin^2 A + \cos^2 A = 1$ (The Pythagorean identity requires both angles be equal.)

In this case, $\frac{\overline{BC}^2}{\overline{AB}^2} + \frac{\overline{BC}^2}{\overline{AB}^2} = 1$

$2\left(\frac{15}{17}\right)^2 \neq 1$

Incorrect.

Bert: sin A = cos B

$\frac{opposite}{hypotenuse} = \frac{adjacent}{hypotenuse}$

$\frac{\overline{BC}}{\overline{AB}} = \frac{\overline{BC}}{\overline{AB}}$

Correct.

Carly: $b = \sqrt{17^2 - 15^2}$

$\overline{AC}^2 + \overline{BC}^2 = \overline{AB}^2$ (Pythagorean's Theorem)

$b^2 + 15^2 = 17^2$

$b^2 = 17^2 - 15^2$

$b = \sqrt{17^2 - 15^2}$

Correct.

Danielle: tan A > 1

$\frac{\overline{BC}}{\overline{AC}} > 1$

$\frac{15}{8} > 1$

Correct.

Three of these observations are correct.

4. **37π**

$(x - h)^2 + (y - k)^2 = r^2$

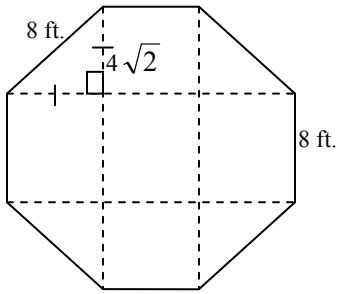
Radius (r) = $\sqrt{37}$

Area = $\pi r^2 = 37\pi$

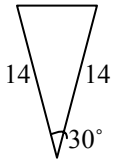
5. Answer: **$-216xy^3$**

$\binom{4}{3}(2x)(-3y)^3 = -216xy^3$

6. Answer: **$128\sqrt{2}$ ft.^{2}**



Area of octagonal engine = Area of four $4\sqrt{2}$ by $4\sqrt{2}$ right triangles + area of four $4\sqrt{2}$ by 8 rectangles + area of one 8 by 8 square = 64 + $128\sqrt{2}$ + 64 = 128 + $128\sqrt{2}$ ft.^{2}



Area of $\frac{1}{12}$ a dodecagon = ½ $14^2 \cdot \sin 30°$

= ¼ 14^2

= 49^2

Area of dodecagon = 49 ft.$^2 \cdot 12$

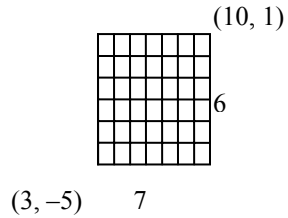
= 588 ft.^{2}

Sum of area of octagon and area of dodecagon

= $(128 + 128\sqrt{2}) + 588$

= $716 + 128\sqrt{2}$ ft.^{2}

7. Answer: **1716**



Possible paths from (3, –5) to (10, 1)

$= \frac{(7+6)!}{7!6!}$

$= \frac{13!}{7!6!}$

$= \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

= 1716

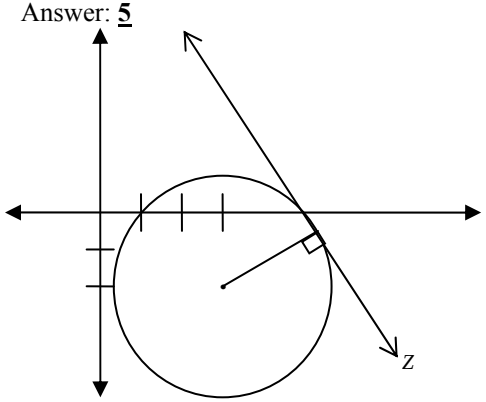
8. Answer: **(3, -2/3)**
(3, –4), (4, 6), (2, –4)
where x-coordinates of points are x_1, x_2, and x_3; y-coordinates of points are y_1, y_2, and y_3.
Centroid (x, y):

x-coordinate = $\frac{x_1 + x_2 + x_3}{3} = \frac{3+4+2}{3} = \frac{9}{3} = 3$

y-coordinate = $\frac{y_1 + y_2 + y_3}{3} = \frac{-4+6+-4}{3} = \frac{-2}{3}$

Centroid $(3, \frac{-2}{3})$

9. Answer: **5**



$y + 2 = ½ (x – 3)$
$2y + 4 = x – 3$
$x = 2y + 7$

$(x – 3)^2 + (y + 2)^2 = 5$
$[(2y + 7)]^2 + (y + 2)^2 = 5$
$4y^2 + 16y + 16 + y^2 + 4y + 4 = 5$
$5y^2 + 20y + 15 = 0$
$y^2 + 4y + 3 = 0$
$(y + 3)(y + 1) = 0$
$y = –3, –1$

(1, –3) , (5, –1)

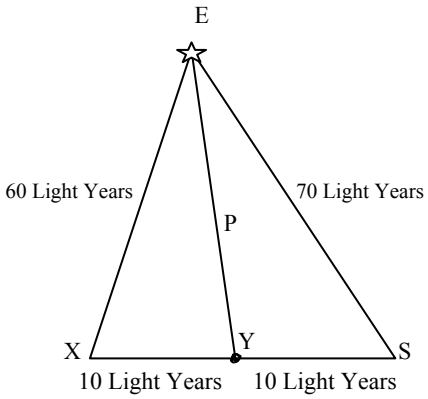
$y + 1 = –2(x – 5)$
$y = –2x + 9$
$b = 9, a = –4$
$a + b = 5$

10. Answer: **4**
Prime palindromes cannot begin in an even number because the entire number will be divisible by 2. Therefore all numbers beginning with 2, 4, and 6 cannot be used to find a prime palindrome. Prime palindromes also cannot begin with a 5 because the entire number will be divisible by 5. Therefore, the remaining numbers are 303, 313, 323, 333, 343, 353, 363, 373, 383, and 393. 303, 363, 333, 393 are divisible by 3; 323 is divisible by 17, and 343 is divisible by 7. The remaining 4 are prime: 313, 353, 373, and 383.

11. Answer: **9 (or 9 seconds)** Plugging in the given coordinates gives the following system of equations:
$36 = a(0) + b(0) + c \quad \Rightarrow \quad c = 36$
$56 = a(1) + b(1) + c \quad \Rightarrow \quad a + b = 20 \quad \Rightarrow \quad a + b = 20$
$70 = a(4) + b(2) + c \quad \Rightarrow \quad 4a + 2b = 34 \quad \Rightarrow \quad \underline{2a + b = 17}$
Subtracting the two equations on the right gives a = -3 and b = 23.

The equation is now $h = -3t^2 + 23t + 36$. When the rocket hits the ground, its height will be 0. So, solve $0 = -3t^2 + 23t + 36$. Factoring gives: $-(3t + 4)(t – 9)= 0$. The rocket lands on the ground at t= 9 seconds.

12. Answer: $\mathbf{5}\sqrt{166}$



Using Stewarts's Theorem by law of cosines:
$60(10)(60) + 70(10)(70) = P(20)P + 10(20)(10)$
$49000 + 36000 = 20P^2 + 2000$
$83000 = 20P^2$
$P^2 = 4150 \qquad P = 5\sqrt{166}$