Rocket City Math League 2004-2005

Apollo Solutions Round 1

1. Answer: **2**
   Plug in x = 1 into the equation, and the value obtained will be the remainder. For the equation $5x^4 - 2x^3 + 7x^2 - 13x + 5$:

   $$5(1)^4 - 2(1)^3 + 7(1)^2 - 13(1) + 5$$
   $$= 5 - 2 + 7 - 13 + 5$$
   $$= 2$$

2. Answer: **45º**
   (a) $(7x - 5) + y = 180$
   (b) $(2x + 5) + y = 90$ (rearranging equation b results in)
   $y = 90 - (2x + 5)$ (plugging y into equation a results in)
   $(7x - 5) + 90 - (2x + 5) = 180$
   $5x - 10 = 90$
   $x = (90 + 10) / 5 = 20$

   Plugging x back into equation a, you get that:
   $y = 180 - 7(20) + 5 = 45º$

3. Answer: **1356_8**
   $750_{10} = \underline{\ ?\ }_8$

   $$\begin{array}{r|l} 8 & 750 \quad 6 \\ 8 & 93 \quad 5 \\ 8 & 11 \quad 3 \\ & 1 \end{array}$$

   Thus, the total number of scientists needed is 1356_8.

4. Answer: **100**
   $S^2 * P = k$ (where k is a constant)

   $$S^2 + P = k$$
   $$10^2 + 4 = k = 400$$

   $$S^2 + P = 400$$
   $$2^2 * P = 4 * P = 400$$
   $$P = 100$$

5. Answer: **3**
   The sum of all three astronaut's computer files = 999

   $3^x + 3^{x+3} + 3^{x+2} = 999$
   $3^x + 3^x(3^3) + 3^x(3^2) = 999$
   $3^x (1 + 3^3 + 3^2) = 999$
   $3^x (1 + 27 + 9) = 999$
   $3^x (37) = 999$
   $3^x = 27$
   $\log_3 3^x = \log_3 27$
   $x = 3$

6. Answer: **2520**
   This question asks for the total number of combinations that can be created with 2 astronauts, 5 technicians, and 3 programmers. (There can be no repetitions)
   $$\frac{10!}{2!\cdot 5!\cdot 3!} = 2520$$

7. Answer: **3**
   Tom Marvolo Riddle has 16 characters of which the "o" is repeated three times, and the "m", "l", "r", and the "d" twice. This gives a total of $\frac{16!}{3!\times 2!\times 2!\times 2!\times 2!}$ = 435891456000. There are 3 zeros at the end.

   Alternate Solution: To get a trailing zero, there must be a factor of 10 in the numerator. There are plenty of 2's in the numerator. Therefore, to find the total number of trailing zeros, count the total number of fives in the numerator. 5, 10, and 15 each have a factor of 5 so there are exactly 3 zeros.

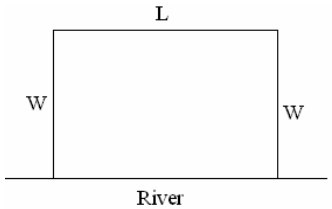
8. Answer: **6**
   $\log_7(m + 1) + \log_7(m - 5)$
   $= \log_7[(m + 1)*(m - 5)]$
   $= \log_7(m^2 - 4m - 5)$

   $\log_7(m^2 - 4m - 5) = 1$
   $7^1 = (m^2 - 4m - 5)$
   $m^2 - 4m - 12 = 0$
   $(m - 6)(m + 2) = 0$
   $m = 6 \cup -2$

   –2 is an extraneous solution therefore m = 6.

9. Answer: **3,200**
   This can be solved by finding the maximum value of a function.

Because the fence only has three sides, L + 2W = 160.



L = 160 – 2W
Area = L * W = (160 – 2W) * W
Area = $160W - 2W^2$
Area = $-2\,(W^2 - 80W)$
Area = $-2(W - 40)^2 + 3200$ (Quadratic function with maximum value at the vertex)
W = 40, L = 80
Therefore, maximum area is W * L = 40 * 80 = 3,200

10. Answer: **36 π**
Rotating this triangle around the x-axis will result in an inner and outer cone. The outer cone will have a height of 7 and a radius of 6 while the inner cone will have a height of 4 and a radius of 6.

Volume of outer cone: $\left(\frac{1}{3}\right)(\pi)(6^2)(7) = 84\pi$

Volume of inner cone: $\left(\frac{1}{3}\right)(\pi)(6^2)(4) = 48\pi$

The total volume created by the rotation of this triangle is:
$84\pi - 48\pi = 36\,\pi$

11. Answer: **4012**

The sum of the first *n* natural numbers is $\frac{n(n+1)}{2}$. The sum of the first *n* squares is $\frac{n(n+1)(2n+1)}{6}$. Dividing the two yields $\frac{3}{(2n+1)}$. Plugging in 2004 gives $S = \frac{3}{(2\times 2004+1)} = \frac{3}{4009}$. Finally $a + b = 3 + 4009 = 4012$.

12. Answer: **169/2744**
Probability of drawing a purple marble is 1/14.
Probability of not drawing a purple marble is 13/14

In order for Xyplon to win on his second turn, he must lose on his first turn and Krunzon must lose on his first turn. The probability Xyplon wins is:

$$\left(\frac{13}{14}\right)\cdot\left(\frac{13}{14}\right)\cdot\left(\frac{1}{14}\right) = \frac{169}{2744}$$