Rocket City Math League 2004-2005
Apollo Solutions Round 3

1. Answer: **729,000**

$(\underline{1}\ \underline{0})^3 = (9\cdot 10)^3 = 729\cdot 1000 = 729{,}000$ combinations

$\underline{2}\ \underline{1}$
$\underline{3}\ \underline{2}$
$\vdots\ \ \vdots$
$\underline{9}\ \underline{0}$

2. Answer: **15**

$$40\% = \frac{x(60\%) + y(10\%)}{25}$$

$1000 = 60x + 10y$
$100 = 6x + y$
$-(25 = x + y)$
$75 = 5x$
$x = 15;\ y = 10$

The rocket scientist uses 15 gallons of the 60% mixture

3. Answer: **5**

Given $3x^2 + 5x - 1 = 0$ where m and n are solutions:

$$\frac{1}{m} + \frac{1}{n} = \frac{n}{mn} + \frac{m}{mn} = \frac{m+n}{mn} = \frac{\text{sum of roots}}{\text{product of roots}} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{-b}{c} = 5$$

4. Answer: **35**

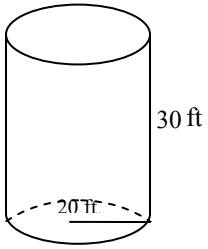
$$S_n = 1 + 2^2 + 3^2 + \ldots + 9^2 + 10^2 = \frac{(10)(11)(21)}{6} = 385$$

$$n = 1 + 2 + 3 + \ldots + 9 + 10 = \frac{(11)(10)}{2} = 55$$

Median = 28th term = 7; Mode = 10; Range = 10 – 1 = 9

$$\text{Mean} = \frac{S_n}{n} = 7;\ A = (\text{median} \times \text{mode}) \div (\text{range} - \text{mean}) = \frac{(7)(10)}{9-7} = 35$$

5. Answer: **20 (or 20 gallons)**



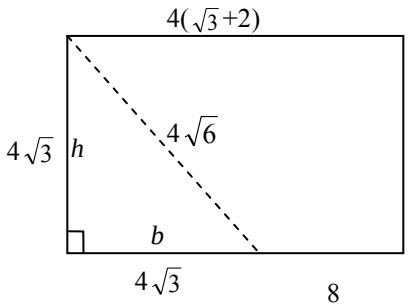
Volume = $\pi r^2 h = 12{,}000\pi$
$r = 20$ ft.
$12{,}000\pi = 400\pi h$
$h = 30$ ft
one gallon = 100π ft.^{2}

SA = $2\pi r^2 + 2\pi rh = 2000\pi$ ft^2
Paint = $2000\pi/100\pi = 20$ gallons

6. Answer: **2920**

$$\frac{365 \text{ earth days} \cdot 8 \text{ octon days}}{1 \text{ earth year} \cdot 1 \text{ earth day}} = \frac{2920 \text{ Octon days}}{1 \text{ earth year}}$$

7. Answer: $\mathbf{40 + 40\sqrt{3} + 4\sqrt{6}}$



b(base) = h(height)
$A = \frac{1}{2}bh = 24$
$b^2 = 48$
$b = h = \sqrt{48} = 4\sqrt{3}$
$A = 24 + 8(4\sqrt{3})$
$P = 4\sqrt{3} + 4\sqrt{3} + 4\sqrt{6} + 16$
$= 8\sqrt{3} + 4\sqrt{6} + 16$
$A + P = 40 + 40\sqrt{3} + 4\sqrt{6}$

8. Answer: **90**

In base 8, the 3-digit numbers are from 100_8 to 777_8. In base 10, the numbers are 64 to 511. The numbers divisible by 5 begin at 65 and end at 510, inclusive.

$A_n = A_1 + 5(n-1)$
$510 = 65 + 5(n-1)$
$n = 90$

9. Answer: **210/73**

Alex (7 days) = 1 rocket; $A_{rate} = \frac{1}{7}$ (rocket per day)

$T_{rate} = \frac{1}{6}$, $J_{rate} = \frac{1}{10}$

$\frac{4}{5}(A_{rate} + T_{rate}) + J_{rate}$

$= \frac{4}{5}\left(\frac{1}{7} + \frac{1}{6}\right) + \frac{1}{10} = \frac{4}{5}\left(\frac{13}{42}\right) + \frac{1}{10} = \frac{26}{105} + \frac{1}{10} = \frac{73}{210}$

Work = Rate * Time

Work = 1 rocket built = $\frac{73}{210} \cdot t$

$t = \frac{210}{73}$

10. Answer: $\left(2 + \frac{5\sqrt{7}}{4}, -\frac{5}{4}\right)$ or $\left(\frac{8 + 5\sqrt{7}}{4}, -\frac{5}{4}\right)$

(6, 1), (–2, 1); c = 4; center (2, 1)

a = 5, b = 3

$\frac{(x-2)^2}{25} + \frac{(y-1)^2}{9} = 1$ and

$(x-2)^2 + (y-1)^2 = 16$

So, $(x-2)^2 = 16 - (y-1)^2$

$\frac{16 - (y-1)^2}{25} + \frac{(y-1)^2}{9} = 1$

$\frac{16}{25} - \frac{(y-1)^2}{25} + \frac{(y-1)^2}{9} = 1$

$(y-1)^2\left(\frac{1}{9} - \frac{1}{25}\right) = \frac{9}{25}$

$(y-1)^2\left(\frac{16}{9}\right) = 9$

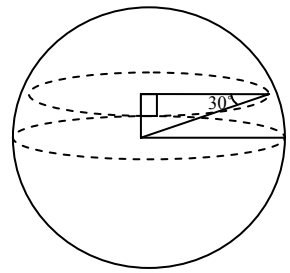
$(y-1)^2 = \pm\frac{9}{4}$

$y = 1 \pm \frac{9}{4} = -\frac{5}{4}$ or $\frac{13}{4}$

Quad IV $y = -\frac{5}{4}$

$x = 2 \pm \frac{5\sqrt{7}}{4}$

11. Answer: $500\pi + 250\pi\sqrt{3}$



Fogg:

circumference of the equator

$= 40{,}000\pi$

Average Speed

$= 40{,}000\,\pi \div 80$

$= 500\,\pi$ (miles per day)

Passé partout:

radius of Great circle

$= \left(\frac{r\sqrt{3}}{2}\right)$

$= \left(\frac{20{,}000\sqrt{3}}{2}\right)$

$2\pi r$ = circumference = $20{,}000\pi\sqrt{3}$

Average Speed = $\frac{20{,}000\pi\sqrt{3}}{80} = 250\pi\sqrt{3}$

Sum: $500\,\pi + 250\pi\sqrt{3}$

12. Answer: $\sqrt{41}$

Since $\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$ and $A \times f(x) = 4A\sin(x) + 5A\cos(x)$, to make these correspond, $4A = \cos(y)$ and $5A = \sin(y)$

Further, $\sin(y)^2 + \cos(y)^2 = 1 = (4A)^2 + (5A)^2 = 41A^2$, $1 = 41A^2$ and $A = \frac{1}{\sqrt{41}}$.

Thus, $\frac{f(x)}{\sqrt{41}} = \cos(y)\sin(x) + \cos(x) + \sin(y) = \sin(x+y)$ and $f(x) = \sqrt{41}\sin(x+y)$.

Since y is a constant angle, the amplitude of $f(x)$ is $\sqrt{41}$.

(The general formula for the amplitude of $a\sin(x) + b\cos(x)$ is $\sqrt{a^2 + b^2}$)