Rocket City Math League 2004-2005

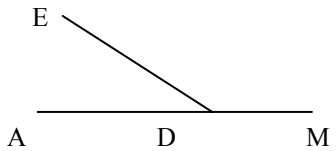
Gemini Solutions

1. Answer: **2/3**
Rearranging the equation to the y-intercept form (y = mx + b):

$$-3y = -2x + 10$$
$$3y = 2x - 10$$
$$y = 2/3x - 10/3$$

2. Answer: **18º**
The sum of the angles in a triangle always equals 180º. The angles in the triangle are xº, (3x + 12) º, and (5x + 6) º:
xº + (3x + 12) º + (5x + 6) º = 180º
9xº + 18º = 180º
9xº = 162º
x = 18º

3. Answer: **65º**



$m\angle ADE + m\angle EDM = 180º$
4x + 1 + 7x + 3 = 180º
11x + 4 = 180º
11x = 176º
x = 16º

Plugging 16º back in for x:
$m\angle ADE$ = 4x + 1 = 4(16º) + 1 = 65º
$m\angle EDM$ = 7x + 3 = 7(16º) + 3 = 115º
The smaller of the two angles is then 65º.

4. Answer: **18º**
The sum of the exterior angles of any polygon equals 360º. Therefore, a regular icosagon that has 20 sides will have an exterior angle with a measure of 360º/20, or 18º.

5. Answer: **8**
The sum of the interior angles of a polygon is 180º (n – 2). Half the sum of the interior angles of a polygon is then 90º (n – 2).
So 90º (n – 2) = 540º, (n – 2) = 6, and n = 8.

6. Answer: **$2\sqrt{6}$**
Altitude (from A to $\overline{BC}$) = 2 (Area of ΔABC)/ $\overline{BC}$
Using Heron's Formula:
S = semi-perimeter = ½ (a + b + c) = ½ (5 + 6 + 7) = 9
Area ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{9(4)(3)(2)}$
$= 6\sqrt{6}$
Altitude = 2 ($6\sqrt{6}$)/6
$= 2\sqrt{6}$

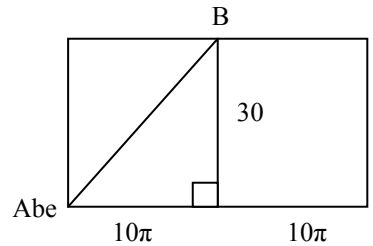
7. Answer: **280**
Number of parallelograms = (7 + 6 + 5 + 4 + 3 + 2 + 1)(4 + 3 + 2 + 1)
= (28)(10)
= 280

8. Answer: **$144\sqrt{3} - 72\pi$**
The area of equilateral ΔABC is $\frac{s^2\sqrt{3}}{4}$, where s is a side of the triangle.

Each side of the triangle is 2(radius of one of the congruent circles) = 2(12) = 24.
$\frac{24^2\sqrt{3}}{4} = 144\sqrt{3}$.

The area of the shaded region is the area of ΔABC – 3(sector of each circle, contained in ΔABC).
The area of the sector of each circle, contained in ΔABC
$= \frac{60°}{360°}\pi \cdot r^2$ (The ratio of 60º to 360º is used; 60º comes from the angle of the sector, which is an angle of the equilateral ΔABC)
$= \frac{1}{6}\pi \cdot 12^2$
$= 24\pi$
Area of shaded region = $144\sqrt{3} - 3(24\pi) = 144\sqrt{3} - 72\pi$

9. Answer: **$10\sqrt{9+\pi^2}$**
Flattening the soda can out results in the following figure and measurements:



The distance between Abe and point B
$= \sqrt{30^2 + (10\pi)^2}$
$= \sqrt{900 + 100\pi^2}$
$= 10\sqrt{9 + \pi^2}$

10. Answer: **Questions was removed**

If you ignore that $\angle ABC = 30^\circ$, then

m$\angle DAC$ = m$\angle ADC$ = 45º

ΔACD is a special right triangle/isosceles triangle (45º–45º–90º)

Therefore, $\overline{AC} = \overline{DC} = 2\sqrt{3}$

$\overline{CB} = \overline{CD} + \overline{BD} = 2\sqrt{3} + 6$

ΔABC = ½ (base)(height) = ½ $(2\sqrt{3} + 6)(2\sqrt{3}) = 6 + 6\sqrt{3}$

ΔACD = ½ (base)(height) = ½ $(2\sqrt{3})(2\sqrt{3}) = 6$

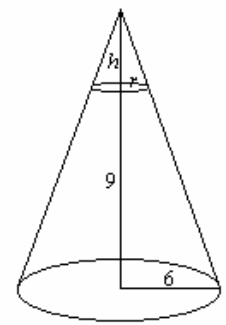
ΔABC = ΔACD + ΔADB

ΔADB = ΔABC – ΔACD

$= (6 + 6\sqrt{3}) - (6)$

$= 6\sqrt{3}$

11. Answer: **$3(3^{1/3})$**



Original volume of Corey = $\frac{1}{3} \cdot \pi \cdot \text{radius}^2 \cdot \text{height}$

$= \frac{1}{3} \cdot \pi \cdot 36 \cdot 9 = 108\pi$

Remaining Volume after frustration = $\frac{8}{9}$ · Original volume of Corey

$= \frac{8}{9} \cdot 108\pi = 96\pi$

Volume of Head = $108\pi - 96\pi = 12\pi$

Volume of Head = $\frac{1}{3}\pi \cdot r^2 \cdot h = 12\pi$

A ratio from similar cones: $\frac{h}{9} = \frac{r}{6}$; $6h = 9r$; $h = \frac{3}{2}r$

Substituting $\frac{3}{2}r$ in for h: $\frac{1}{3}\pi \cdot r^2 \cdot \frac{3}{2}r = 12\pi$, and $r = 2\sqrt[3]{3}$

Solving for h: $\frac{3}{2} \cdot 2\sqrt[3]{3} = 3\sqrt[3]{3}$

12. Answer: **6π**

In regular tetrahedron ABCD: $\overline{DE}$ and $\overline{BE}$ are medians, $\overline{DF}$ is the height of the tetrahedron, and $\overline{FG}$ is the radius of the inscribed sphere, and s is the side of the tetrahedron (6).

$\overline{DE}$ is the height of equilateral ΔACD, and $\overline{BE}$ is the height of equilateral ΔABC.

Therefore, $\overline{DE} = \overline{BE} = \frac{1}{2}\sqrt{3}s$

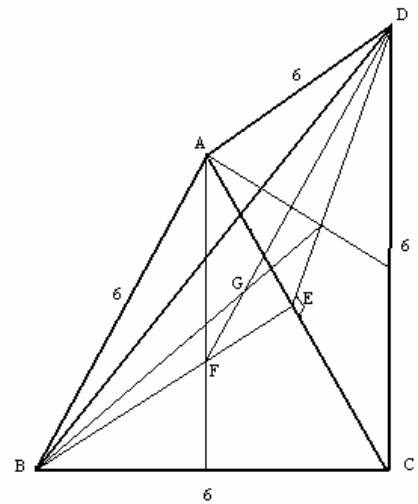
$\overline{FE} = \frac{1}{3}\overline{BE} = \frac{1}{6}\sqrt{3}s$

$\overline{DF}^2 = (\frac{1}{2}\sqrt{3}s)^2 - (\frac{1}{6}\sqrt{3}s)^2 = \frac{1}{4}3s^2 - \frac{3}{36}s^2 = \frac{2}{3}s^2$

$\overline{DF} = \frac{\sqrt{6}}{3}s$

From ΔBDE: $\frac{\overline{DG}}{\overline{GF}} = \frac{2 \cdot 3}{1 \cdot 2} = \frac{3}{1}$

radius of inscribed sphere = $\overline{GF} = \frac{1}{4}height = \frac{1}{4} \cdot \frac{\sqrt{6}}{3}s = \frac{\sqrt{6}}{12}s$

$s = 6$, and $r = \frac{\sqrt{6}}{12}s = \frac{\sqrt{6}}{2}$

Surface area = $4 \cdot \pi \cdot r^2 = 4 \cdot \pi \cdot 3/2 = 6\pi$