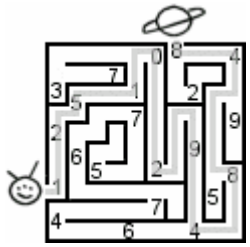
Rocket City Math League 2004-2005

Mercury Solutions Round 3

1. Answer: **44**

The path that Tsol must take is shown below:



Add together the numbers that the path crosses:

$$1 + 2 + 5 + 1 + 0 + 2 + 9 + 4 + 8 + 4 + 8 = 44$$

2. Answer: **Sna**

To find which Madiator finished in the fastest time, convert each time to a decimal:

Tram:	$3 + \sqrt{2} \approx 4.41...$
Nait:	$^{133}/_{35} = 3.8$
Sna:	$2.3837 + 1.298 = 3.6817$
Itra:	$(2.6 - 0.24)^2 = 5.5696$
Am:	$(3 + 8 \div 10) = 3.8$

Sna has the shortest time, 3.6817 hours. Therefore, Sna wins the Madiator World Cup.

3. Answer: **481**

Space R' Us must sell 60% of 800 spacesuits, or 480 spacesuits to pay off its debts in full. In order to gain a positive profit, one additional spacesuit must be sold.

4. Answer: **588**

The astronauts will be in hypersleep for a total of 30% of the trip and out of hypersleep for 70% of the trip. 70% of 70 years is 49 years, or 588 months.

5. Answer: **102**

The following system of equations can be written using the given information. Let the Galaxy Milky Way = M, the Galaxy Andromeda = A, and the Galaxy Archon = C.

$M = 2A - 80$	→	$A = {}^1/_2 M + 40$
$C = M - 218$	→	$C = M - 218$
$M + A + C = 622$	→	$M + {}^1/_2 M + 40 + M - 218 = 622$

Solving for the third equation gives M = 320. Thus, C = M - 218 = 320 - 218 = 102.

6. Answer: **1**

First factor the equation by grouping, then unfoil:

$$x^3 + 3x^2 - 4x - 12 = x^2 (x + 3) - 4 (x + 3) = (x^2 - 4) (x + 3)$$
$$x^3 + 3x^2 - 4x - 12 = (x + 2) (x - 2) (x + 3)$$

To find the positive root, set (x + 2) (x - 2) (x + 3) equal to 0. This produces the roots of {-2, 2, and –3}. The only positive root is 2. So if the combined weight of E.T.'s head and helmet is 2 pounds, and E.T.'s head is 1 pound, then his helmet also weighs 1 pound.

7. Answer: **60**

If the flight path is perpendicular to y = -2x, then its equation is $y = {}^1/_2 x$. The y-intercept is insignificant because the question asks for the number of miles flown, or the difference between the starting point and the stopping point. Since 2 minutes equals 120 seconds, $y = {}^1/_2 (120) = 60$ miles flown.

8. Answer: **3528**

Let A be the number of NASA's astronauts. The following equation can be written using the given information:

$${}^1/_4 A - {}^1/_3 ({}^1/_4 A) - {}^5/_6 ({}^2/_3) ({}^1/_4 A) = 98$$
$${}^1/_4 A - {}^1/_{12} A - {}^5/_{36} A = 98$$
$$9A - 3A - 5A = 3528$$
$$A = 3528$$

9. Answer: **985**

When each pair of consecutive numbers is added together, the new summation produced is:

-1 -1 -1 -1 - ... -1 + 1969

There are 1968 ÷ 2 = 984 pairs of the original numbers, so there are 984 (-1)'s. Therefore, 1969 - 984 = 985.

10. Answer: **9**

C is 0 as in a typical multiplication problem. Also, since L times the number RCML gives RCML, L must be 1. Thus, R is 5, M is 3, and P is 9.

R + C + M + L = 5 + 0 + 3 + 1 = 9

11. Answer: **28**

-7 -10 -12 -12 -9 -2 10	original sequence
-3 -2 0 3 7 12	differences between two above terms
1 2 3 4 5	etc.
1 1 1 1	etc.

Continue the above patterns:

-7 -10 -12 -12 -9 -2 10 28
-3 -2 0 3 7 12 18
1 2 3 4 5 6
1 1 1 1 1

So the next term is 28.

12. Answer: **10**

Each locker is changed by an astronaut numbered with one of the number's factors. (Ex. Locker 12 will be changed by Astronauts 1,2,3,4,6, and 12). If the locker number has an odd number of factors, the locker will change position an odd number of times and will remain open. The numbers from 1 to 100 with an odd number of factors are the perfect squares (1,4,9,16,25,36,49,64,81,100). As a result, 10 lockers will end up open.